Table 1.28. Comparison of Effective Areas at Low Pressure by Two Methods

Nominal areas of assemblies compared, sq. in.	Ratio of effective areas	
	Computed from measured dimensions	From direct balancing test
0·2 and 0·125 (*a*)	1·59927	1·59923
0·2 and 0·125 (*b*)	1·59931	1·59933
0·125 and 0·125	1·00002	1·00001
0·05 and 0·05	1·00002	1·00004
0·02 and 0·02	1·00005	1·00004

DETERMINATION OF VARIATION OF EFFECTIVE AREA AT ELEVATED PRESSURES

The method recently developed at the N.P.L. (which may conveniently be named the 'similarity' method) is completely independent of the high-pressure mercury manometer, and depends on the application of a principle of similarity to the distortions of a pair of piston-cylinder assemblies of the same nominal dimensions but constructed of materials whose elastic constants are in a known ratio to one another. Provided that a number of conditions which are considered below are satisfied, the distortions of two such assemblies will remain proportional to one another throughout a range of pressure appropriate to the particular design of assembly. This state of affairs leads very directly to a method of determining the absolute value of the distortion of each assembly. Denoting by A_P and B_P the effective areas of the two assemblies at a high pressure P, and A_0 and B_0 the corresponding areas at zero pressure we may write

$$A_P = A_0\left[1 + \Delta(\alpha, P)\right]; \qquad B_P = B_0\left[1 + \Delta(\beta, P)\right] \quad \ldots\ldots (1.40)$$

where the quantities $\Delta\,(\alpha, P)$ and $\Delta\,(\beta, P)$ represent the effects of distortion under the applied pressure, and are functions of P and of the constants α and β representing the elastic constants of the two materials. Ignoring small quantities of the second order—since the Δ values are very small compared with unity—this leads to the expression

$$\frac{A_P}{B_P} = \frac{A_0}{B_0}\left[1 + \Delta\,(\alpha, P) - \Delta\,(\beta, P)\right] \ldots\ldots (1.41)$$

As already discussed above, the ratio A_P/B_P may be determined to a high degree of accuracy by balancing the two assemblies directly against one another, in fact apart from trivial corrections the ratio is simply the ratio of the loads on the two balances when these are in equilibrium.

It is now necessary to consider under what conditions the distortions of the two assemblies may be taken to be proportional to one another. In the distortion of a system as complex as a practicable piston assembly, two independent elastic moduli will be involved. The expansion of the cylinder under internal pressure is largely determined by the modulus of rigidity of the material provided the external diameter is reasonably large compared with the diameter of the bore (Newitt 1940). In assemblies similar to Fig. 1.23 *a*, the piston

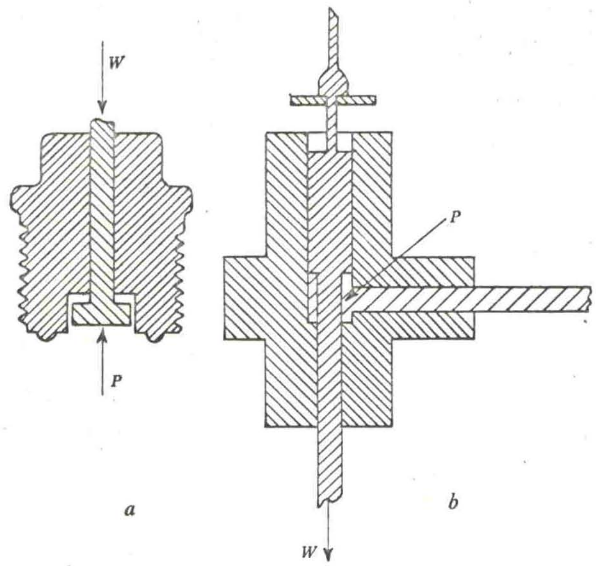

Fig. 1.23. Two Types of Piston–Cylinder Assembly

a Simple piston. *b* Differential piston.
The effective area of the assembly is the ratio of the load W to applied pressure P when the balance is in equilibrium.

is subjected to a compressive stress along its length due to the load and the pressure acting at its ends, and to a varying stress at right angles to its axis along the flanks due to the fluid pressure in the gap. The distortion of the piston will thus involve both the modulus of rigidity and the other independent modulus of the material. Strict similarity between the distortions of the two pistons therefore requires both moduli to be in the same ratio, or in other words that the values of Poisson's ratio for the two materials should be substantially the same.

Certain conditions must also be imposed on the mechanical forms of the components. For instance, the forms of the internal bores of the cylinders and of the pistons must be closely similar in contour, in order to ensure that the distribution of pressure in the gap between piston and cylinder is the same in the two assemblies. The similarity condition also requires that the widths of the gaps between piston and cylinder for the two assemblies at zero pressure should be inversely proportional to the elastic moduli of the materials. It is also clearly necessary that the elastic moduli should be